

CONTRIBUTION OF ARYABHATA TO GEOMETRY AND ASTRONOMY

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Abstract

Here, we are discussing the grand and glorious contributions of Āryabhata-I (476 A.D.-550 A.D. expected), Varāhamihir (Born 505 A.D.), Bhaskara (Born 525 A.D.), Brahmagupta (598 A.D. - 669 A.D. expected), Mahāvīrācārya (round about 850 A.D.), Āryabhata-II (950 A.D.), Bhāskara-II (1114 A.D. - 1185 A.D. expected), Nārāyana Pandita (1356 A.D.) and others to the knowledge of Geometry on the soil of India.

With the fifth century after Christ we reach surer ground of Āryabhata-I, who dominated the Indian mathematical world. Here we have gone through the thirty three verses given in the Ganitapāda of Āryabhata-I which speaks of his high level attainments and gives us the formulae for obtaining the area of the triangle, area of a circle, circumference of a circle, volume of a sphere, volume of a pyramid and value of π .

But the formulae for the volume of sphere and that of pyramid propounded by Āryabhata-I were found to be incorrect. It is a matter of high appreciation that Āryabhata-I conceived the concept of irrationality of π thirteen hundred years ago, when Lambert rediscovered that π is an irrational number and provided its formal proof in 1761 A.D.

Keywords : Āryabhata-I, Āryabhata-II, Bhāskara-I, Bhāskara-II, Varāhamihir, Mahāvīrācārya, Ganitapāda, Circumference, Irrational number.

1. Introduction

In this paper, we go through valuable mathematical formulae and theorems propounded by Āryabhata-I, Brahmagupta and others in Ancient India and see how many of them have been rediscovered after a lapse of centuries in Europe. We can see that the "Theorem of the Square of the Hypotenuse in a rational right angle triangle" was in wide use in Śulbasūtras with the impressive initiative of Baudhāyana (800 B.C.). We also find trace of this theorem in Taittirīya Samhitā (3000 B.C.) and Śatapatha Brāhmaṇa (2000 B.C.). Here it has been concluded that the "Theorem of the Square of the Hypotenuse in a rational right angle triangle" should be associated with the name of Baudhāyana (800 B.C.) instead of the name of Pythagoras (540 B.C.).

Āryabhata-I laid the foundation stone of Algebra in 5th century A.D. on the soil of Ancient Bihar. He devised a novel method to express large number. His contribution is advancing the knowledge of Geometry, Mensuration, Square root, Cube root and Celestial sphere also deserve praises. It is a fact that Āryabhata-I was the first to deduce that the earth is round and that it also spins on its own axis, creating day and night. He declared that the moon is dark and it shines only because of reflection of sunlight. Solar and lunar eclipses, he believed occurred not because Rahu gobbled the Sun and the moon as Hindu mythology claimed but because of the shadows east the earth on the moon (lunar eclipse) and the moon obstructing the light of the Sun (Solar eclipse).^[1] Without any fear being a Brahmin Āryabhata-I declared his result what he had found, that solar and lunar eclipses were not because Rahu gobbled the Sun and the moon. This was strictly against Hindu mythology. Hence, so many Astronomers did not agree with the opinions of Āryabhata-I. But today Āryabhata-I's result is found to be cent-percent correct. This leads us to the conclusion that Āryabhata-I was a revolutionary thinker and true mathematician. He, however, believed in the geo-centric concept of the Universe that the earth is the centre of the Universe. To explain the "erratic" movements of some planets, he, like the Greek King Ptolemy, made use of "epicycles". But his method was superior to Ptolemy's.^[2]

In Mathematics, Āryabhata-I contributions are equally valuable. He gave the value of π as 3.14146....., claiming for the first time that it was an approximation. Āryabhata-I also deals with other aspects of Mathematics and astronomical calculations, namely geometry, mensuration, square root, cube root, progression and celestial sphere. It was in appreciation of Āryabhata-I's contribution to Astronomy and Mathematics that Indian first satellite was named Āryabhata-I.

2. Different Values of π .

2.1. Value of π

The value of π , i.e. the ratio of the circumference to the diameter have been taken to be equal to 3.00, 3.004, 3.0883, 3.0885, 3.16049, 2.99, 3.029, 3.047 and 3.3 in different Śulbasūtras. It is to be noted that the symbol π was first used by William Jones to denote the ratio of the circumference to the diameter in a circle. We see that the value of π has been taken to be equal to 3 in Jaina's work Suryaprajñapti whereas most of the Jaina's works like the Jyotiskarandaka and the Jambūdvīpsamāsa of Umāsvāti records the value of π to be equal to $\sqrt{10}$. Further we find that Virsenā, the author of Dhavata Tikā on the Satkhandāgama has given a better value of π . It has been taken to be equal to $\frac{355}{113}$, a close approximation for π .

2.2. Āryabhata-I 's Value of π

Āryabhata-I has discussed the ratio of the circumference to the diameter of a circle in the following verse :

चतुरधिकं शतमष्टगुणं द्वाषष्टिस्थिता सहास्राणाम्।

अयुतद्वयविष्कम्भस्यासन्नो वृत्तपरिणाहः॥

Translation

"Adding four to hundred, multiplying by eight and then adding these to sixty two thousands, we get the approximate value of the circumference of a circle of diameter twenty thousands."

$$\begin{aligned}\text{Here, we find that the circumference of the circle} &= 104 \times 8 + 62000 \\ &= 832 + 62000 \\ &= 62832 \text{ (Approximately)}\end{aligned}$$

And its diameter = 20,000.

$$\begin{aligned}\therefore \frac{\text{Circumference}}{\text{Diameter}} &= \frac{62832}{20000} \\ \text{i.e. } \pi &= \frac{62832}{20000} = \frac{3927}{1250} \\ &= 3.1416\end{aligned}$$

The modern value of $\pi = 3.14159\ldots$

Thus, we find that the Āryabhata-I's value is a very close approximation to the modern value of π and among the ancients it is the most accurate. The use of the word 'Āsanna' leads us to conclude that Āryabhata-I was not satisfied even with this accurate value of π correct to four places of decimal and was also conscious of the possibility of more closer approximate value of π .

3. Relation between the chord and the radius in a circle

The second part of the ninth verse in Āryabhatīya is given below

परिधेष्पदभागज्या विष्कम्भार्थेन सा तुल्या॥

Translation

"The chord of the sixth part of the circumference is equal to the semi-diameter (of the circle)".

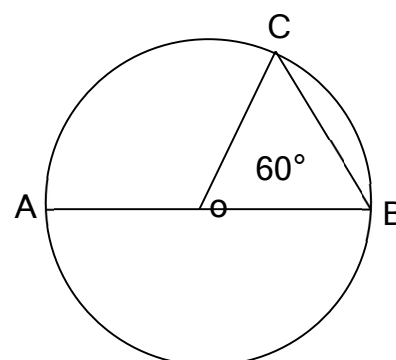
Let us take a circle with its centre at the point O and the straight line AOB as one of its diameters.

Further, we take the arc BC to be one sixth of the whole circumference of the circle so that the angle subtended by the arc BC at the centre O will be 60° .

$$\text{i.e. } \angle BOC = 60^\circ$$

In $\triangle OBC$, We have,

$OB = OC = \text{Radius of the circle}$



oo

$$\therefore \angle OBC = \angle OCB = \frac{1}{2} \times (180^\circ - 60^\circ)$$

$$\text{i.e. } \angle OBC = \angle OCB = 60^\circ$$

Thus, the $\triangle OBC$ is equilateral so that,

$$OB = OC = BC$$

i.e. Chord BC = radius

This establishes Āryabhata-I's rule given here.

Here we can see that Āryabhata-I while giving the above rule, might have thought of the ratio of the circumference to the diameter of the circles. As, the chord of an arc is always less than its arc, so we have

$$\text{arc BC} > \text{chord BC}$$

$$\text{i.e. arc BC} > OB$$

$$\text{i.e. } 6 \times \text{arc BC} > 6 \times OB$$

$$\text{i.e. Circumference} > 3 \times AB$$

$$\text{i.e. } \frac{\text{Circumference}}{AB} > 3$$

$$\text{i.e. } \frac{\text{Circumference}}{\text{Diameter}} > 3$$

Thus, the ratio of the circumference to the diameter is greater than 3. Most probably, this result was known to Āryabhata-I and as such he might have thought of obtaining the exact value of ratio, which had been vexed problem for the ancient people.

Of course, Āryabhata-I succeeded in deriving the value of π $\left(= \frac{3927}{1250} = 3.1416 \right)$,

which bears high degree of accuracy. He has used the value of π in obtaining the formulae for the area of circle^[3] and also for the volume of a sphere.^[4] He has also used the value of π in the preparation of a scientific table for the differences of sines^[5] and in calculating the dimension of the earth and its atmosphere.^[6] There are some of the examples found in the Āryabhatīya, which confirm the fact that Āryabhata-I, has used the value of π several times.

The value of π determined by Āryabhata-I, has been adopted by many Indian as well as foreign mathematicians. Its use, for the first time, is believed to have been found in the works of Lalla and it is expressed in the following manner. If we multiply the circumference by 625 and divide the result by 3927, we get the corresponding radius.^[7]

This yields the value of π as $\frac{3927}{1250} = 3.1416$.

Āryabhata-II^[8] also adopts the value of π with a slight difference as given below.

$$\pi = \frac{21600}{6876} = 3.1416.....,$$

which is a close approximation to that of Āryabhata-I. On the other hand, we see that Brahmgupta, Śrīdharācārya, Mahāvīracārya and other Indian mathematicians adopted different values of π viz. $\sqrt{10}$, $\frac{22}{7}$ etc. but they were also not unaware of its exact value yielded by Āryabhata-I.

Again, we see that Chinese Astronomer Tsu Chung - Chih (C.470) derived the value of π .

$$\left(= \frac{355}{113} = 3.141592..... \right)$$

Of course it is a more correct value of π . But the process adopted by the Chinese Astronomer in obtaining the value of π is quite different from the process adopted by Āryabhata-I.^[9]

We can find the trace of π on the soil of India long before the time of Āryabhata-I and even before the birth of the Greek Civilization. In Ancient India, we go through problems like, 'squaring of the circles' and vice versa which led the discovery of the value π . We can find such problem in Rigveda and Samhitās in connection with the construction of sacrificial altars but we do not find the direct mention of the value of π in these works. The earliest definite information regarding the value of π is found in the Śulbasūtras. Here we get different values of π such as 2.99, 3, 3.004, 3.0083, ..., 3.0885, 3.029, 3.088, 3.16049 etc. Of course, these values of π are not as accurate as its modern value is 3.141592.....

Nilkantha^[10], one of the commentators of the Āryabhatīya and C.N.S. Lengar, on the basis of the commentary of Nilkantha^[11], concluded that Āryabhata-I did not only derive the most accurate value of π , but also pointed out that π is an irrational number. It is a matter of high appreciation that Āryabhata-I conceived the idea of the irrationality of π thirteen hundred years ago, when Lambert rediscovered that π is an irrational number and provided its formal proof in 1761 A.D.

4. Area of a Circle

People in ancient India were familiar of deriving the area of a circle as given below :

$$\text{Area of circle} = \text{Circumference} \times d/4.$$

Where, d is the length of a diameter of the circle.

This formula for determining the area of a circle is clearly mentioned in Umāsvātis (150 B.C.). Tattvārthā d hīg amabhāsyā, though no discussion on the proof of the formula is found here. In Āryabhatīya, we find that the area of a circle is discussed in the following verse :

समपरिणाहस्यार्थं विष्कम्भार्धहतमेव वृत्तफलम् ॥
(Āryabhatīya 11.7)

Translation

"Half the circumference (of a circle) multiplied by half the diameter is undoubtedly the area of the circle."

Here, we have,

$$\text{Area of a circle} = \frac{1}{2}(\text{Circumference}) \times \frac{1}{2}(\text{diameter})$$

In our modern notations, we may write it in the following way :

$$\begin{aligned}\text{Area of a circle} &= \frac{1}{2}\pi d \times \frac{1}{2}d \\ &= \frac{1}{4}\pi d^2 \\ &= \pi \left(\frac{d}{2}\right)^2 \\ &= \pi r^2\end{aligned}$$

Where, d is the diameter of the circle and r is the radius of the circle.

This is a correct formula which we still use it all over the world.

In this connection, we may mention that Āryabhata-I might have been inspired for this rule by the Śulbasūtras ^[12] as well as from the Jaina mathematics ^[13] (Where it is believed to have been used in connection with the problems like 'circling the square and vice versa'). We do not find the rule to be expressed explicitly in the earlier works, while Āryabhata-I mentioned it clearly and correctly. Some of the later mathematicians, viz. Brahmagupta ^[14], Āryabhata-II ^[15] and others gave the rule in a slight different form, while the same form of the rule was adopted by Bhāskara-II. Bhāskara-II says, "the circumference multiplied by the diameter divided by four gives the area of a circle." ^[16]

Āryabhata-I is most probably the first mathematician who used the term 'Viskambhārdha' (equivalent of semi-diameter) for the radius. Euclid used the term 'Distance' in this sense. Boethius (C.510), however, used the equivalent of semi diameter but after Āryabhata-I (476 A.D.). From India it seems to have passed over to Arabia and thence to Europe. ^[17] The word 'Semi-diameter' was used by the two mathematician of the west, they are Fibonacci (1220 A.D.) and Nemorarius (C.1225). We find the word radius, first of all, in the works of Ramus (C. 1569 A.D.), which was opted by Vieta (C.1590 A.D.) and others. Hence we see that Āryabhata-I stands as a pioneer in this respect.

5. Theorem on the "Square of the Hypotenuse" in Āryabhatīya

यश्चैव भुजावर्गः कोटिवर्गश्चः कर्णवर्गः सः ॥

(Āryabhata Ganitapāda .17)

Translation

"What is the sum of squares of the perpendicular is the square of the hypotenuse."

The two terms 'Bhujā' and 'Koti' refer to the two sides of a right angle triangle, perpendicular to each other. 'Koti' is that side which is perpendicular to 'Bhujā', no matter it is upright or horizontal. The third side of the triangle is termed as 'Karna' the hypotenuse.

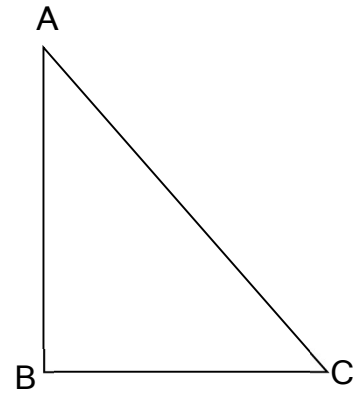
Here, we take a right angled triangle ABC in which 'Koti' is either AB or BC. If AB is considered as 'Koti' BC is 'Bhujā' and vice-versa. The third side AC of the right-angled triangle ABC is termed as the 'Karna'.

Now, according to the rule stated above,
We have,

$$AB^2 + BC^2 = AC^2$$

$$\text{i.e. } AC^2 - AB^2 = BC^2$$

$$\text{and } AC^2 - BC^2 = AB^2$$



We see that Āryabhata-I did not give the proof of this theorem. But it does not mean that its proof was unknown to him. It was the general practice of the ancient Hindu mathematicians to express only the results without their proof. The credit of the discovery of this theorem generally goes in the name of the celebrated Greek philosopher Pythagoras 6th century B.C. But in earlier discussion, we have seen that Baudhāyana propounded this theorem in Śulbasūtras in the 8th century B.C.

Bhāskarā cārya has also dealt this theorem in Lilāvati. It is evident from the following verse:

तत्कृत्योर्योगपदं कर्णो दोः कर्णवर्गयोर्विवरात्।

मूल कोटिः कोटिश्रुतिकृत्योरतरात् पदं बाहुः॥

(Lilāvati CXLII)

Translation

"In right angled triangle, square root of the sum of squares of the base and the altitude is the hypotenuse."

In Bhāskarā cārya Bijaganita, the rational for the theorem is given.

दोः कोट्यन्तरवर्गेण द्विध्नो घातः समन्वितः।

वर्गयोगसमः स स्याद् द्वयोरव्यक्तयोर्यथा॥

(Bijaganita, 129)

Translation

"Twice the product of the Bhujā and koti combined with the square of their difference will be equal to the sum of their squares, just as it is so for two algebraical quantities".

Here, we take a right angled triangle ABC and draw AD perpendicular to the base BC from the point A in the adjoining figure. We clearly see that triangle ADB, triangle ADC and triangle ABC are similar to each other.

From similar triangle ABC and triangle ABD,
We have,

$$\frac{AB}{BC} = \frac{BD}{AB}$$

$$\text{i.e. } BD \cdot BC = AB^2$$

$$\text{i.e. } BD = \frac{AB^2}{BC}$$

Again, from similar triangle ABC and triangle ACD,
We have,

$$\frac{AC}{BC} = \frac{DC}{AC}$$

$$\text{i.e. } DC \cdot BC = AC^2$$

$$\text{i.e. } DC = \frac{AC^2}{BC}$$

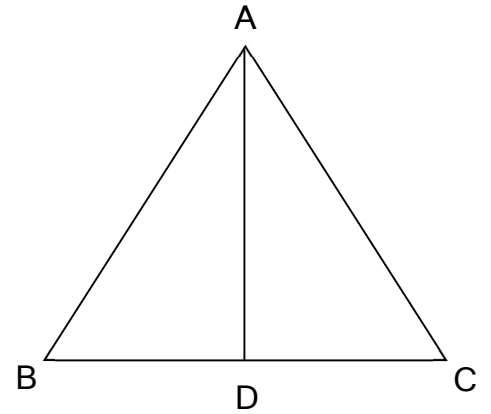
Now adding BD and DC, we have,

$$BD + DC = \frac{AB^2}{BC} + \frac{AC^2}{BC}$$

$$\text{or, } BC = \frac{AB^2 + AC^2}{BC}$$

$$\text{or, } BC^2 = AB^2 + AC^2$$

This proves the theorem of the 'Square of the Hypotenuse' in the right angled triangle ABC.



6. Conclusion

Thus, we come to the conclusion that Āryabhaṭīya-I laid the foundation – stone of Algebra, in 5th century A.D. on the soil of Ancient Bihar. He also devised a novel method to express large numbers such as 100,000,000,000 in words. He developed this particular method to write unwieldy numbers in poetic form. Concise but somewhat difficult to grasp Āryabhaṭīya-I also deals with other aspects of Mathematics and astronomical calculations, namely geometry, mensuration, square root, cube root, progression and celestial sphere. It was in appreciation of Āryabhaṭīya-I's contributions to Astronomy and Mathematics that India's first satellite was named Āryabhaṭīya-I.

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